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NCAR GLOBAL GENERAL CIRCULATION MODEL OF THE ATMOSPHERE

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ABSTRACT

This paper describes a model of the general circulation of the earth's atmosphere which has been developed and experimented with, since 1964, at the National Center for Atmospheric Research (NCAR), Boulder, Colo. A distinguishing feature of the NCAR model is that the vertical coordinate is height rather than pressure, though hydrostatic equilibrium is maintained in the system. In fact, the dynamical framework of the model is very similar to the one proposed by L. F. Richardson in 1922.

Various physical processes in the atmosphere, such as energy transfer due to solar and terrestrial radiation, small-scale turbulence and convection, etc., are incorporated in the model. An explicit prediction of the moisture field is avoided. Instead, it is assumed that the atmosphere is completely saturated by water vapor. Thus, the release of latent heat of condensation can be computed. In addition to a description of the model, the equations for the zonal mean and eddy energy are presented. Finally, a baroclinic stability analysis of the model is made in order to gain an insight into the finite-difference formulation of the present model. Long term (over 100 days) numerical integrations are being performed successfully with a two-layer version of the present model. Details of finite-difference schemes and the results of numerical calculations will be described in a separate article.

"Perhaps some day in the dim future it will be possible to advance the computations faster than the weather advances and at a cost less than the saving to mankind due to the information gained. But that is a dream." [From *Weather Prediction by Numerical Process* by L. F. Richardson, 1922.]

1. INTRODUCTION

The purpose of this paper is to describe a model of the general circulation of the earth's atmosphere which has been developed and experimented with at the National Center for Atmospheric Research (NCAR), Boulder, Colo.

Following the successful development of numerical weather prediction techniques at the Institute for Advanced Study in Princeton, Phillips [34] first attempted a numerical simulation of the general circulation. His work was based on a two-level quasi-geostrophic model in a zonally periodic channel on a β-plane. The experiment demonstrated that certain gross properties of the atmospheric motion including the energy-transformation processes can be simulated numerically in the model.

To remove the quasi-geostrophic assumption employed in Phillips' model, Smagorinsky [43] formulated a two-level model by using the Eulerian equations of motion within a spherical zonal strip. He succeeded in performing long-term integrations of the Eulerian equation model.

Following these earlier attempts, more elaborate types of general circulation experiments using the Eulerian equations of motion and thermodynamic equations were designed and carried out by Leith [18], Mintz [28], and A. Arakawa [1]. In these models various physical processes in the atmosphere, such as energy transfer caused by solar and terrestrial radiation, small-scale turbulence and convection, the release of latent heat, etc., are simulated. Various features of general circulation experiments up to summer 1965 are discussed in an article by Gavrilin [10] and also in a report prepared by the Panel on Weather and Climate Modification [31]. The results of experiments based on more sophisticated models have recently been published by Smagorinsky, Manabe, and Holloway [44] and by Manabe, Smagorinsky, and Strickler [22].

In the beginning of 1964, after studying the characteristics of various general circulation experiments thus far reported, we decided to perform general circulation experiments at NCAR using a model which is somewhat different from those described so far. The purpose of this undertaking is to construct a numerical model of the atmosphere which is simple but general enough that increasingly refined methods of simulating physical processes in the atmosphere, such as the transport processes of heat and momentum, can be incorporated as they are developed, and flexible enough that scientists from diverse fields can test their hypotheses.

A distinguishing feature of the NCAR general circulation model is that the vertical coordinate is height rather than pressure, though hydrostatic equilibrium is maintained in the system. The use of pressure as the vertical coordinate in the hydrostatic system is more common, since the continuity equation is reduced to a diagnostic equation without a time derivative term. However, this advantage is offset by a disadvantage in the handling of the lower boundary conditions of the system, since the earth's surface usually does not coincide with a constant pressure surface. In order to eliminate this difficulty, Phillips [35] introduced the so-called "σ-coordinate" system which has been adopted in the Smagorinsky-Manabe and Mintz-Arakawa models. Some of the advantages of using height as the vertical coordinate are that the prognostic equations have simpler forms than those in the σ-system,* and the lower boundary conditions may easily be formulated.

The dynamical framework of the NCAR model is therefore very similar to the one proposed by Richardson [39]. Besides the difference in the use of the vertical coordinate, the NCAR model includes various physical processes in the atmosphere which have been formulated somewhat differently from those incorporated in other general circulation models.

In Section 2, the dynamical framework of the model is discussed. The basic equations are all written in spherical coordinates in order to treat motions in both hemispheres. In Section 3, the upper and lower boundary conditions are presented. In Section 4, the heating/cooling produced by solar and terrestrial radiation is discussed. One of the most important heat sources in the atmosphere is the latent heat released by condensation in clouds. However, to simplify the formulation of the first version of the model, an explicit prediction of the moisture field is avoided. Instead we assume that the atmosphere is completely saturated by water vapor. Therefore, the release of latent heat takes place in the region of ascending motion. The rate of condensation, however, is altered by multiplying a parameter. In Section 5, the energy equations of the model are derived, since little work has been published concerning the derivation of equations for the zonal mean and eddy energy of various forms in the z-system. In Section 6, a baroclinic stability analysis of the model is presented for the case of a two-layer model in order to gain an insight into the finite-difference formulation of the present model. In Section 7, a brief summary is made of numerical calculations performed successfully with a two-layer version of the present model.

2. BASIC EQUATIONS

Let us define the following symbols which appear frequently in the text.

φ=latitude (positive northward)
λ=longitude (positive eastward)
z=height
t=time
$\mathbf{V}$=horizontal velocity
u, v=longitudinal and latitudinal components of $\mathbf{V}$
w=vertical velocity
p=pressure
ρ=density
T=temperature
$\alpha=1/\rho$, specific volume
g=acceleration due to gravity
a=mean radius of the earth
Ω=angular velocity of the earth
$f=2\Omega \sin \varphi$, Coriolis parameter
R=gas constant for dry air
c_p=specific heat for dry air at constant pressure
c_v=specific heat for dry air at constant volume $(=c_p-R)$
$\gamma=c_p/c_v$
$\kappa=R/c_p$, Poisson's constant for dry air
$\mathbf{F}$=frictional force per unit volume
F_λ, F_φ=longitudinal and meridional components of $\mathbf{F}$
Q=rate of heating per unit mass.

In spherical coordinates, λ, φ, and z, the longitudinal and meridional equations of motion appropriate to the problem are

*It was reported in [45] that computations of pressure gradient on the σ-coordinate created some difficulties over steep sloping terrain. In the equations of motion in the σ-system, the pressure gradient is computed from two terms. When these terms are evaluated by finite differences, the truncation errors of two terms are not necessarily consistent with each other and consequently a large error can be produced.

$$\frac{\partial(\rho u)}{\partial t}=-\nabla\cdot(\rho u\mathbf{V})-\frac{\partial}{\partial z}(\rho uw)-\frac{1}{a\cos\varphi}\frac{\partial p}{\partial\lambda}+\left(f+\frac{u}{a}\tan\varphi\right)\rho v+F_\lambda, \quad (2.1)$$

$$\frac{\partial(\rho v)}{\partial t}=-\nabla\cdot(\rho v\mathbf{V})-\frac{\partial}{\partial z}(\rho vw)-\frac{1}{a}\frac{\partial p}{\partial\varphi}-\left(f+\frac{u}{a}\tan\varphi\right)\rho u+F_\varphi, \quad (2.2)$$

where, for a scalar quantity A,

$$\nabla\cdot A\mathbf{V}=\frac{1}{a\cos\varphi}\left[\frac{\partial(Au)}{\partial\lambda}+\frac{\partial}{\partial\varphi}(Av\cos\varphi)\right]. \quad (2.3)$$

The mathematical approximations employed for the derivation of (2.1) and (2.2) have been discussed by Phillips [37].

The mass continuity equation is

$$\frac{\partial\rho}{\partial t}=-\nabla\cdot(\rho\mathbf{V})-\frac{\partial}{\partial z}(\rho w). \quad (2.4)$$

The thermodynamic equation may be expressed by

$$\frac{d\rho}{dt}=\frac{1}{\gamma RT}\frac{dp}{dt}-\frac{\rho}{c_pT}Q \quad (2.5)$$

where

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla+w\frac{\partial}{\partial z} \quad (2.6)$$

and

$$\mathbf{V}\cdot\nabla=\frac{u}{a\cos\varphi}\frac{\partial}{\partial\lambda}+\frac{v}{a}\frac{\partial}{\partial\varphi}. \quad (2.7)$$

It is assumed that the atmosphere is in hydrostatic equilibrium, i.e.,

$$\frac{\partial p}{\partial z}=-\rho g. \quad (2.8)$$

The equation of state for an ideal gas is

$$p=\rho RT. \quad (2.9)$$

By substituting equation (2.8) into the left-hand side of (2.4) and then integrating the resulting equation from z to the top of the atmosphere, the height of which is denoted by z_T, we obtain the well-known pressure tendency equation

$$\frac{\partial p}{\partial t}=B+g\rho w-g\int_z^{z_T}\nabla\cdot(\rho\mathbf{V})dz \quad (2.10)$$

where $B=\partial p/\partial t$ evaluated at $z=z_T$. In deriving (2.10) it is assumed that

$$w=0 \text{ at } z=z_T. \quad (2.11)$$

This boundary condition will be used throughout the present work.

The vertical velocity w is calculated in such a way that hydrostatic equilibrium is always maintained between p and ρ. In other words, the vertical velocity w must be evaluated to satisfy both (2.4) and (2.5) together with the hydrostatic condition (2.8). The vertical derivative of w may be obtained from (2.4) in the following form:

$$\frac{\partial w}{\partial z}=-\nabla\cdot\mathbf{V}-\frac{1}{\rho}\frac{d\rho}{dt}. \quad (2.12)$$

By substituting (2.5) into (2.12) one finds that

$$\frac{\partial w}{\partial z}=-\nabla\cdot\mathbf{V}-\frac{1}{\gamma p}\frac{dp}{dt}+\frac{1}{c_pT}Q. \quad (2.13)$$

With the aid of (2.8), we rewrite (2.10) in the form:

$$\frac{dp}{dt}=B+J \quad (2.14)$$

where

$$J=\mathbf{V}\cdot\nabla p-g\int_z^{z_T}\nabla\cdot(\rho\mathbf{V})dz. \quad (2.15)$$

By substituting (2.14) into (2.13) and integrating the resulting equation from $z=0$ to some height z, we obtain

$$w=-\int_0^z\nabla\cdot\mathbf{V}dz-\frac{1}{\gamma}\int_0^z\frac{1}{p}(B+J)dz+\frac{1}{c_p}\int_0^z\frac{Q}{T}dz \quad (2.16a)$$

where we assumed that

$$w=0 \text{ at } z=0, \quad (2.17)$$

which is the lower boundary condition on w used throughout this work.

In (2.16a), the term B, the pressure tendency at the top, has been left as an unknown. This will be determined in such a way that (2.16a) satisfies the upper boundary condition (2.11). This leads to

$$B=\frac{\frac{1}{\gamma}\int_0^{z_T}\frac{J}{p}dz-\frac{1}{c_p}\int_0^{z_T}\frac{Q}{T}dz+\int_0^{z_T}\nabla\cdot\mathbf{V}dz}{-\frac{1}{\gamma}\int_0^{z_T}\frac{dz}{p}}. \quad (2.16b)$$

An equation for w similar to (2.16) was derived by Richardson [39]. (See also [7] and [50].) In his derivation, however, it was assumed that dp/dt vanishes at the top of the atmosphere $z=\infty$ instead of the condition (2.11).

The independent variables of the system are λ, φ, z, and t. The dependent variables are the longitudinal and latitudinal components u and v of velocity which are predicted by (2.1) and (2.2); and the pressure p, which is predicted by (2.10) with the aid of (2.16). The vertical motion is computed from the diagnostic equation (2.16). Temperature and density are computed from the hydrostatic relationship (2.8) with the use of the gas law (2.9).

The longitudinal and meridional components F_λ and F_φ of the eddy viscous force may be expressed in the following forms:

$$\left.\begin{aligned}F_\lambda&=\frac{\partial\tau_\lambda}{\partial z}+\rho K_{MH}\left[\nabla^2u-\frac{u}{a^2\cos^2\varphi}-\frac{2\sin\varphi}{a^2\cos^2\varphi}\frac{\partial v}{\partial\lambda}\right]\\ F_\varphi&=\frac{\partial\tau_\varphi}{\partial z}+\rho K_{MH}\left[\nabla^2v-\frac{v}{a^2\cos^2\varphi}+\frac{2\sin\varphi}{a^2\cos^2\varphi}\frac{\partial u}{\partial\lambda}\right]\end{aligned}\right\} \quad (2.18)$$

where K_{MH} represents the horizontal kinematic eddy viscosity and

$$\nabla^2 \equiv \frac{1}{a^2 \cos\varphi} \frac{\partial}{\partial\varphi}\left(\cos\varphi \frac{\partial}{\partial\varphi}\right) + \frac{1}{a^2 \cos^2\varphi} \frac{\partial^2}{\partial\lambda^2}. \quad (2.19)$$

The last three terms in (2.18) represent the lateral frictional force. These terms are derived by assuming that the fluid is incompressible and that horizontal variations of density are neglected. Also we assumed that the contributions from terms containing w can be neglected (see for example Goldstein [11]). In the actual computations, we ignored the last two terms in (2.18) because they are, in general, small in magnitude compared with that of the Laplacian term. The longitudinal and latitudinal components τ_λ and τ_φ of Reynold's stress may be expressed by

$$\tau_\lambda = \rho K_{MV} \frac{\partial u}{\partial z}; \quad \tau_\varphi = \rho K_{MV} \frac{\partial v}{\partial z} \quad (2.20)$$

where K_{MV} represents the vertical kinematic eddy viscosity. The magnitudes of K_{MH} and K_{MV} will be discussed in a separate paper, together with the results of computations.

The heating rate Q in (2.16a) consists of the following three parts:

$$Q = Q_a + Q_c + Q_d. \quad (2.21)$$

The terms Q_a and Q_c are the heating terms from radiational sources and the release of latent heat by condensation, respectively. These terms will be discussed in Section 4. The term Q_d is the rate of heating/cooling due to eddy diffusion which may be expressed by

$$Q_d = -\frac{1}{\rho} \frac{\partial h}{\partial z} + c_p K_{TH} \nabla^2 \theta \quad (2.22)$$

where K_{TH} represents the horizontal kinematic thermal diffusivity and θ denotes the potential temperature

$$\theta = T\left(\frac{p_0}{p}\right)^\kappa \quad (2.23)$$

with $p_0 = 1000$ mb. The quantity h denotes the vertical flux of sensible heat which may be expressed by

$$h = -\rho c_p K_{TV}\left(\frac{\partial\theta}{\partial z} - \gamma_{CG}\right) \quad (2.24)$$

where K_{TV} denotes the vertical kinematic thermal diffusivity and γ_{CG} is called the counter-gradient constant. Deardorff [5] has suggested the use of a small counter-gradient constant in (2.24) in order to explain observed upward heat flux when the vertical gradient of θ is slightly greater or equal to zero. Therefore, the counter-gradient constant shifts the point at which heat flux is either upward or downward. The heating processes other than by eddy diffusion will be described in Section 4. Concerning the value of γ_{CG} in (2.24), Deardorff [5] reported that $\gamma_{CG} = 0.65 \times 10^{-5}$°C. cm.$^{-1}$ However, he said that the value of γ_{CG} will be larger than that given above at levels where cloudiness prevails. In the present calculations, it is assumed that $\gamma_{CG} = 5 \times 10^{-5}$ °C. cm.$^{-1}$

3. BOUNDARY CONDITIONS

At the top of the atmosphere, $z = z_T$, we assume in the upper boundary conditions that the vertical motion, the surface stress components, and the sensible heat flux all vanish. Thus,

$$w = \tau_\lambda = \tau_\varphi = h = 0 \text{ at } z = z_T. \quad (3.1)$$

The effects of the planetary frictional (Ekman) layer are included in the model by introducing the vertical eddy (subgrid scale) exchange processes for momentum and heat. The effects of the surface boundary (Prandtl) layer are simulated by the lower boundary conditions for the model in order to evaluate surface stress and sensible heat flux at the earth's surface. The thickness of the surface boundary layer is on the order of 50 m. It is assumed that the eddy fluxes of momentum and heat are constant with height in the surface boundary layer. The horizontal stress components and the vertical flux of sensible heat in the surface boundary layer are expressed in the customary fashion as follows:

$$\left.\begin{aligned} \tau_\lambda &= C_D \rho_s u_s V_s \\ \tau_\varphi &= C_D \rho_s v_s V_s \\ h &= -c_p C_D \rho_s (T_s - T_g) V_s \end{aligned}\right\} \text{ at } z = z_s \quad (3.2)$$

evaluated at the anemometer level $z = z_s$ located usually near $z = 10$ m. In (3.2) the subscripts s for the quantities refer to those at the anemometer level, T_g represents the temperature of the earth's surface, C_D is the drag coefficient, and

$$V_s = \sqrt{u_s^2 + v_s^2}.$$

The surface stress and the sensible heat flux in the planetary frictional layer have been expressed by (2.20) and (2.24) respectively. It will be assumed that the eddy fluxes of momentum and heat are continuous at the interface of the two boundary layers. Thus, the lower boundary conditions of the problem are expressed in the forms:

$$\left.\begin{aligned} K_{MV} \frac{\partial u}{\partial z} &= C_D u_s V_s \\ K_{MV} \frac{\partial v}{\partial z} &= C_D v_s V_s \\ K_{TV}\left(\frac{\partial\theta}{\partial z} - \gamma_{CG}\right) &= C_D (T_s - T_g) V_s \\ w &= 0 \end{aligned}\right\} \text{ at } z = z_s \quad (3.3)$$

where the boundary condition that the vertical motion vanishes at the earth's surface is added to the set (3.3).

Similar boundary conditions were used by Kasahara [14] in his numerical experiments on hurricane formation.

4. HEATING/COOLING PROCESSES IN THE MODEL ATMOSPHERE

All energy of atmospheric motion is ultimately derived from incoming solar radiation. If directly reflected radiation is ignored, the atmosphere receives from the sun 2.0 cal. cm.$^{-2}$ min.$^{-1}$, which is known as the solar constant [13]. Since the mean temperature of the atmosphere does not change appreciably from one year to the next, the amount of energy received by the earth must be sent back to outer space. A large portion of the outgoing energy is emitted back in the form of infrared radiation.

Referring to the heating rate Q of (2.21), let us divide the heating term due to radiational sources Q_a into two parts,

$$Q_a = Q_{as} + Q_{ae} \tag{4.1}$$

where Q_{as} denotes the rate of heating due to absorption of the solar insolation by water vapor and Q_{ae} is the rate of heating/cooling due to infrared radiation.

Let F_{as} represent the flux of energy of the direct solar radiation absorbed by water vapor in a vertical column extending from height z to the top of the atmosphere. The heating rate Q_{as} is then computed from

$$Q_{as} = -\frac{1}{\rho}\frac{\partial F_{as}}{\partial z}. \tag{4.2}$$

According to Mügge and Möller [29], London [19], and McDonald [24], it is shown that

$$F_{as} = a^*[x(z) \sec \zeta]^{0.3} \cos \zeta \tag{4.3}$$

where $x(z)$ denotes the pressure-corrected precipitable water (gm./cm.^{2}) in a unit column defined by

$$x = \int_z^{z_T} \left(\frac{p}{p_0}\right) \rho_w dz + x_\infty \tag{4.4}$$

in which ρ_w is the density of water vapor and p_0 is a standard pressure (1013 mb.). To obtain a value of the water vapor content in the atmosphere above $z = z_T$, we assume that the mixing ratio is constant with height. Then, the pressure-corrected precipitable water (gm./cm.^{2}) above $z = z_T$ is given approximately by

$$x_\infty = \frac{1}{2}\left(\frac{p_T}{p_0}\right)\frac{0.622 e_T}{g} \cdot 10^3 \tag{4.5}$$

where e_T, the vapor pressure measured in units of mb. at the tropopause, is determined by the pressure p_T and the temperature at $z = z_T$ with assumed relative humidity. According to London [19], the value of average relative humidity at the tropopause is about 60 percent. We adopted this value for the actual computations.

In formula (4.3), ζ denotes the sun's zenith angle which is expressed by

$$\cos \zeta = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos h \tag{4.6}$$

where φ is the latitude, δ is the sun's declination, and h is the sun's hour angle counting from 0 deg. at noon (1 hr. corresponds to 15 deg.). The value of δ for any time of year can be found in a nautical almanac.

In (4.3), a^* is a constant and is given by $a^* = 0.172$ after Mügge and Möller [29], and $a^* = 0.149$ after McDonald [24]. The difference between the two values is, according to Manabe and Möller [21], due to the fact that McDonald did not take into account the contributions from the absorption bands at 2.7, 3.2, and 6.3μ. Therefore, we adopted the value of Mügge and Möller [29].

The empirical formula (4.3) is applied for clear sky conditions only. The amount of F_{as} is considerably altered by the presence of clouds in the atmosphere. In the first version of our model, we will consider neither the effects of clouds nor the absorption of solar insolation by atmospheric gases other than water vapor. Manabe and Strickler [23] discussed in detail a way in which those effects are taken into account in the radiation calculations for general circulation experiments.

Next, we shall consider the rate of cooling by infrared radiation. Let $F\uparrow(z)$, $F\downarrow(z)$, and T be the total upward and downward infrared radiation fluxes and the temperature at the height z under a clear sky. Then it can be shown that

$$F\uparrow(z) = \sigma T_g^4 + \int_{T_g}^{T} R([\zeta(T') - \zeta(T)], T')dT' \tag{4.7}$$

$$F\downarrow(z) = -\int_{T}^{T_\infty} R([\zeta(T) - \zeta(T')], T')dT' \tag{4.8}$$

where T' is a dummy variable, T_g is the temperature of the earth's surface, and T_∞ denotes the temperature at infinity which is assumed to be absolute zero.

$$R(\zeta, T) = \int_0^\infty \frac{\partial B_\nu}{\partial T}(1 - \tau_f(\zeta, T))d\nu, \tag{4.9}$$

and

$$\zeta(T) = \int_0^{z(T)} \frac{p}{p_0}\sqrt{\frac{T_0}{T}}\, \rho_a dz. \tag{4.10}$$

Here ρ_a is the density of the gas which absorbs radiation. Both pressure and temperature effects on absorption are taken into account through the use of "reduced" optical depth as defined by (4.10). Here, B_ν is the Planck function; τ_f is the transmission function. σ is the Stefan-Boltzmann constant [1.1696×10^{-7} gm. cal./(cm.^{2} day deg.^{4})]. ν is the wave number.

Elsasser and Culbertson [8] compiled tables of R as a function of effective optical path lengths and temperature for water vapor, carbon dioxide, and ozone, major absorption gases in the atmosphere. A computer program has been completed by Sasamori [42] to evaluate the fluxes $F\uparrow(z)$ and $F\downarrow(z)$ by a numerical quadrature of equations (4.7) and (4.8) utilizing the tables of R tabulated in [8].

The rate of heating/cooling Q_{ae} due to the divergence of the net infrared radiation flux can be obtained from

$$Q_{ae}=-\frac{1}{\rho}\frac{\partial}{\partial z}(F\uparrow-F\downarrow). \tag{4.11}$$

One of the most important heat sources in the atmosphere is the latent heat released by condensation in clouds. This heating rate is denoted by Q_c in (2.21). As mentioned in the Introduction, an explicit prediction of the moisture field is avoided. Instead we assume that the atmosphere is completely saturated by water vapor. Thus, the release of latent heat will take place in the region of ascending motion and that heating rate is expressed by

$$Q_c=-E_f Lw\frac{\partial q_s}{\partial z} \tag{4.12}$$

where L is the latent heat of condensation ($=2.5\times10^{10}$ erg gm.$^{-1}$), and q_s denotes the saturation specific humidity. Since the actual atmosphere is not saturated by water vapor, the present procedure may overestimate the amount of release of latent heat in the model atmosphere. To reduce the rate of condensation, we introduced in (4.12) an efficiency factor E_f, and it is assumed that $0<E_f\leq 1$. The value of E_f which is used in the actual computations will be discussed in a separate article.

5. ENERGY EQUATIONS

As an approach to the study of the energetics of the general circulation, it is now customary to resolve the field of variables into the zonal mean and the departures from it. The energy equations in the pressure system have been discussed by numerous authors, but relatively few studies have been made in the z-system. In this section, we discuss the energy equations in the z-system following a general outline given by Miller [27] and van Mieghem [25].

We define the zonal average, denoted by a bar, for any variable A by

$$\overline{A}=\frac{1}{2\pi}\int_0^{2\pi}Ad\lambda,\ A=\overline{A}+A'',\ \overline{A''}=0. \tag{5.1}$$

We also introduce, as done by Reynolds [38], the density weighted mean, denoted by ^, which is defined by

$$\hat{A}=\overline{\rho A}/\overline{\rho},\ A=\hat{A}+A',\ \overline{\rho A'}=0. \tag{5.2}$$

For example, it is shown that the mean value of the kinetic energy becomes

$$\tfrac{1}{2}\overline{\rho\mathbf{V}^2}=\tfrac{1}{2}\overline{\rho}\hat{\mathbf{V}}^2+\tfrac{1}{2}\overline{\rho\mathbf{V}'^2}. \tag{5.3}$$

This relation shows the partition of the kinetic energy into two types, which will be referred to as zonal kinetic energy and eddy kinetic energy.

The application of the averaging operator to the continuity equation (2.4) yields

$$\frac{\partial\overline{\rho}}{\partial t}+\overline{\nabla}_3\cdot(\overline{\rho}\hat{\mathbf{V}}_3)=0 \tag{5.4}$$

where

$$\overline{\nabla}_3\cdot(\overline{\rho}\hat{\mathbf{V}}_3)=\frac{\partial(\overline{\rho}\hat{v}\cos\varphi)}{a\cos\varphi\partial\varphi}+\frac{\partial(\overline{\rho}\hat{w})}{\partial z}. \tag{5.5}$$

Averaging of the momentum equations (2.1) and (2.2) yields

$$\frac{\partial}{\partial t}(\overline{\rho}\hat{u})=-\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}[(\overline{\rho}\hat{u}\hat{v}+\overline{\rho u'v'})\cos\varphi]$$
$$-\frac{\partial}{\partial z}[\overline{\rho}\hat{u}\hat{w}+\overline{\rho u'w'}]+\left(f+\frac{\hat{u}}{a}\tan\varphi\right)\overline{\rho}\hat{v}+\frac{\tan\varphi}{a}\overline{\rho u'v'}+\overline{F}_\lambda, \tag{5.6a}$$

and

$$\frac{\partial}{\partial t}(\overline{\rho}\hat{v})=-\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}[(\overline{\rho}\hat{v}\hat{v}+\overline{\rho v'^2})\cos\varphi]-\frac{\partial}{\partial z}[\overline{\rho}\hat{v}\hat{w}+\overline{\rho v'w'}]$$
$$-\left(f+\frac{\hat{u}}{a}\tan\varphi\right)\overline{\rho}\hat{u}-\frac{\tan\varphi}{a}\overline{\rho u'^2}-\frac{1}{a}\frac{\partial\overline{p}}{\partial\varphi}+\overline{F}_\varphi. \tag{5.6b}$$

Multiplying (5.6a) by $\hat{u}$ and (5.6b) by $\hat{v}$ and using (5.4), we obtain the zonal kinetic energy equation in the form

$$\frac{\partial\overline{K}}{\partial t}+\overline{\nabla}_3\cdot(\overline{K}\hat{\mathbf{V}}_3)+\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}[(\hat{u}\overline{\rho u'v'}+\hat{v}\overline{\rho v'^2})\cos\varphi]$$
$$+\frac{\partial}{\partial z}[\hat{u}\overline{\rho u'w'}+\hat{v}\overline{\rho v'w'}]=-C(\overline{K}+\overline{P},K')-\frac{\hat{v}}{a}\frac{\partial\overline{p}}{\partial\varphi}+\hat{\mathbf{V}}\cdot\overline{\mathbf{F}}, \tag{5.7}$$

where

$$\overline{K}=\tfrac{1}{2}\overline{\rho}\hat{\mathbf{V}}^2 \tag{5.8}$$

denotes the zonal kinetic energy per unit volume and

$$C(\overline{K}+\overline{P},K')=-\overline{\rho u'v'}\frac{\cos\varphi}{a}\frac{\partial}{\partial\varphi}\left(\frac{\hat{u}}{\cos\varphi}\right)$$
$$-\overline{\rho v'^2}\frac{\partial\hat{v}}{a\partial\varphi}+\frac{\tan\varphi}{a}\hat{v}\overline{\rho u'^2}-\overline{\rho u'w'}\frac{\partial\hat{u}}{\partial z}-\overline{\rho v'w'}\frac{\partial\hat{v}}{\partial z}. \tag{5.9}$$

The first term of equation (5.7) represents the local rate of change in the zonal kinetic energy per unit volume. The second term is the flux of $\overline{K}$ due to the mean meridional circulation. The next two terms may be interpreted as the flux of eddy stress due to the mean motion. The terms on the right-hand side are the rate of production (or destruction) of kinetic energy of the mean motion which will be discussed later.

Multiplying (5.4) by gz and using the hydrostatic equation for the zonal mean flow, we obtain the potential energy equation for the zonal mean flow in the form

$$\frac{\partial\bar{P}}{\partial t}+\bar{\nabla}_3\cdot(\bar{P}\hat{\mathbf{V}}_3)=-\hat{w}\frac{\partial\bar{p}}{\partial z} \tag{5.10}$$

where $\bar{P}$ denotes the zonal potential energy defined by

$$\bar{P}=g\bar{\rho}z. \tag{5.11}$$

By adding (5.7) and (5.10), we obtain the equation for the sum of the kinetic and potential energy $\bar{K}+\bar{P}$, which will be referred to as the *zonal mechanical energy*.

$$\frac{\partial}{\partial t}(\bar{K}+\bar{P})+\bar{\nabla}_3\cdot[(\bar{K}+\bar{P}+\bar{p})]\hat{\mathbf{V}}_3+\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}$$
$$\times[(\hat{u}\overline{\rho u'v'}+\hat{v}\overline{\rho v'^2})\cos\varphi]+\frac{\partial}{\partial z}[\hat{u}\overline{\rho u'w'}+\hat{v}\overline{\rho v'w'}]$$
$$=-C(\bar{K}+\bar{P},K')+C(\bar{I},\bar{K}+\bar{P})+\hat{\mathbf{V}}\cdot\bar{\mathbf{F}}, \tag{5.12}$$

where

$$C(\bar{I},\bar{K}+\bar{P})=\bar{p}\bar{\nabla}_3\cdot\hat{\mathbf{V}}_3. \tag{5.13}$$

The equation for eddy kinetic energy can be obtained by averaging the kinetic energy equation for the *total* motion and subtracting (5.7) from the resulting equation. Here we write the final equation without going through intermediate steps:

$$\frac{\partial K'}{\partial t}+\bar{\nabla}_3\cdot(K'\hat{\mathbf{V}}_3)+\bar{\nabla}_3\cdot(\overline{p\mathbf{V}'_3})+\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}$$
$$\times\left[\frac{1}{2}(\overline{\rho u'^2v'}+\overline{\rho v'^3})\cos\varphi\right]+\frac{\partial}{\partial z}\left[\frac{1}{2}(\overline{\rho u'^2w'}+\overline{\rho v'^2w'})\right]$$
$$=C(\bar{I},K')+C(\bar{K}+\bar{P},K')+\overline{\mathbf{V}\cdot\mathbf{F}}-\hat{\mathbf{V}}\cdot\bar{\mathbf{F}}, \tag{5.14}$$

where

$$K'=\tfrac{1}{2}\overline{\rho\mathbf{V}'^2} \tag{5.15}$$

denotes the eddy kinetic energy and

$$C(\bar{I},K')=\overline{p\left(\nabla\cdot\mathbf{V}'+\frac{\partial w'}{\partial z}\right)}. \tag{5.16}$$

The equation for internal energy I, defined by

$$I=c_v\rho T, \tag{5.17}$$

can be derived from the thermodynamic equation (2.5) together with (2.4) and (2.9) as follows:

$$\frac{\partial I}{\partial t}+\nabla\cdot(I\mathbf{V})+\frac{\partial}{\partial z}(wI)=-p\left[\nabla\cdot\mathbf{V}+\frac{\partial w}{\partial z}\right]+\rho Q. \tag{5.18}$$

By applying the averaging operator (5.1) to (5.18), we obtain the internal energy equation for the zonal mean motion:

$$\frac{\partial\bar{I}}{\partial t}+\bar{\nabla}_3\cdot(\bar{I}\hat{\mathbf{V}}_3)+\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}[(c_v\overline{\rho T'v'})\cos\varphi]+\frac{\partial}{\partial z}(c_v\overline{\rho T'w'})$$
$$=-C(\bar{I},K')-C(\bar{I},\bar{K}+\bar{P})+\overline{\rho Q} \tag{5.19}$$

where

$$\bar{I}=c_v\bar{\rho}\hat{T} \tag{5.20}$$

denotes the zonal internal energy. Here the first two terms on the right-hand side of (5.19) have already been defined by (5.16) and (5.13).

The sum of equations (5.12), (5.14), and (5.19) becomes

$$\frac{\partial}{\partial t}(\bar{K}+\bar{P}+K'+\bar{I})+\bar{\nabla}_3\cdot[(\bar{K}+\bar{P}+K'+\gamma\bar{I})\hat{\mathbf{V}}_3]$$
$$+\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}[\{\hat{u}\overline{\rho u'v'}+\hat{v}\overline{\rho v'^2})+\frac{1}{2}(\overline{\rho u'^2v'}+\overline{\rho v'^3})$$
$$+c_p\overline{\rho T'v'}\}\cos\varphi]+\frac{\partial}{\partial z}[(\hat{u}\overline{\rho u'w'}+\hat{v}\overline{\rho v'w'})$$
$$+\frac{1}{2}(\overline{\rho u'^2w'}+\overline{\rho v'^2w'})+c_p\overline{\rho T'w'}]=\overline{\mathbf{V}\cdot\mathbf{F}}+\overline{\rho Q}. \tag{5.21}$$

In (5.21) we used the following relationships:

$$\bar{p}=R\bar{\rho}\hat{T}$$
$$\bar{I}+\bar{p}=(c_v+R)\bar{\rho}\hat{T}=\gamma\bar{I}$$
$$\overline{pv'}=\overline{R\rho(\hat{T}+T')v'}=R\overline{\rho T'v'}$$
$$c_v\overline{\rho T'v'}+\overline{pv'}=c_p\overline{\rho T'v'}.$$

If there is no source of energy and no dissipation, then the right-hand side of (5.21) vanishes and the total energy $\bar{K}+\bar{P}+\bar{I}+K'$ is conserved in the closed system. The first term on the left of (5.21) represents the local rate of change of total energy per unit volume. The second term represents the divergence of the transport of the energy $(\bar{K}+\bar{P}+\gamma\bar{I}+K')$ by the mean meridional circulation. The rest of the terms represent the divergence of the transport of eddy energy by eddy motion.

In (5.14) and (5.21), the terms $(\overline{\mathbf{V}\cdot\mathbf{F}}-\hat{\mathbf{V}}\cdot\bar{\mathbf{F}})$ and $\hat{\mathbf{V}}\cdot\bar{\mathbf{F}}$ represent the dissipation of eddy kinetic energy and the zonal mechanical energy, respectively. Therefore, these terms must be negative, namely

$$\overline{\mathbf{V}\cdot\mathbf{F}}<\hat{\mathbf{V}}\cdot\bar{\mathbf{F}}<0.$$

The energy source function $\overline{\rho Q}$ must be positive, since $\overline{\mathbf{V}\cdot\mathbf{F}}$ is negative if a steady state is maintained.

The function $C(\alpha,\beta)$ defined by (5.9), (5.13), and (5.16) represents the transformation of energy from the form of energy indicated by α into that by β. Note that $C(\alpha,\beta)=-C(\beta,\alpha)$. For example, $C(\bar{K}+\bar{P},K')$ denotes the conversion of the zonal mechanical energy $\bar{K}+\bar{P}$ into the eddy kinetic energy K'. The function $C(\bar{I},\bar{K}+\bar{P})$ denotes the conversion of the zonal internal energy into the zonal mechanical energy. It is difficult to measure this quantity directly from atmospheric data because the evaluation of divergence is involved. However, as often performed in the computation of the vertical motion field (e.g., Panofsky [32]), one may use the approximation

$$\bar{\nabla}_3\cdot\hat{\mathbf{V}}_3\approx-\frac{\hat{w}}{\bar{p}}\frac{\partial\bar{p}}{\partial z}.$$

Thus one obtains

$$C(\bar{I},\ \overline{K}+\overline{P}) \doteq -\bar{\rho}\hat{w}\hat{T}\left(\frac{R}{\bar{\rho}}\frac{\partial \bar{\rho}}{\partial z}\right). \tag{5.22}$$

Similarly, if one introduces the approximation

$$\nabla\cdot\mathbf{V}'+\frac{\partial w'}{\partial z}\approx-\frac{w'}{\bar{\rho}}\frac{\partial \bar{\rho}}{\partial z}$$

then one finds

$$C(\bar{I},K')=-\overline{\rho w'T'}\left(\frac{R}{\bar{\rho}}\frac{\partial \bar{\rho}}{\partial z}\right). \tag{5.23}$$

For a nearly adiabatic lapse rate, the quantity $(R/\bar{\rho})\times(\partial\bar{\rho}/\partial z)$ can be approximated by $-R(g/\gamma)(\bar{\rho}/\bar{p})$.

Figure 1 shows the scheme of energy transformation. In this figure, each of the boxes represents a type of energy. These boxes are connected by the three transformation functions. The net heating of the atmosphere $\overline{\rho Q}$ results in a generation of zonal internal energy $\bar{I}$. The zonal mechanical energy $\overline{K}+\overline{P}$ and the eddy kinetic energy K' are continuously drained out by dissipation processes, denoted by $\hat{\mathbf{V}}\cdot\hat{\mathbf{F}}$ and $\overline{\mathbf{V}\cdot\mathbf{F}}-\hat{\mathbf{V}}\cdot\hat{\mathbf{F}}$.

In the following we shall present an estimate of the magnitude of the three transformation functions $C^*(\overline{K}+\overline{P},\ K')$, $C^*(\bar{I},\ \overline{K}+\overline{P})$ and $C^*(\bar{I},\ K')$. Here the asterisk indicates the quantity which is integrated vertically throughout the depth of the atmosphere and averaged over the Northern Hemisphere for a period of several months. This estimate is based on many studies concerning the evaluation of energy transformation functions. The reader may refer to a useful review article by Oort [30]. The quantity $C^*(\overline{K}+\overline{P},\ K')$ is approximately identical to C_K defined by Lorenz [20]. The dominant term of $C^*(\overline{K}+\overline{P},\ K')$ is the vertical integral of the first term in the expression $C(\overline{K}+\overline{P},\ K')$ given by (5.9), and the significance and evaluation of this integral have been discussed by Kuo [16], Starr [47], Wiin-Nielsen, Brown, and Drake [54], and others. From those data we estimate the magnitude of $C^*(\overline{K}+\overline{P},\ K')$ to be on the order of -0.3 watt m.$^{-2}$ The value is negative so that the eddies supply kinetic energy to the zonal flow to maintain it against dissipation. Thus we put the arrow (fig. 1) pointing from K' to $\overline{K}+\overline{P}$ indicating the general direction of the "flow" of energy.

The terms $C^*(\bar{I},\ K')$ and $C^*\ (\bar{I},\ \overline{K}+\overline{P})$ correspond to those of C_E and C_Z, respectively, as defined in [20]. These relationships are

$$C^*(\bar{I},\ K')\doteq C_E/\gamma,\ C^*(\bar{I},\ \overline{K}+\overline{P})\doteq C_Z/\gamma,$$

where $\gamma=c_p/c_v\doteq1.4$, C_E denotes the conversion from eddy available potential energy to eddy kinetic energy, and C_Z denotes the conversion from zonal kinetic energy to zonal available potential energy.

Based on data analyses by Saltzman and Fleisher [41], Wiin-Nielsen [53], and Krueger, Winston, and Haines [15], we estimate the magnitude of $C^*(\bar{I},K')$ to be on the order of 2.5 watt m.$^{-2}$ This means that the zonal internal energy $\bar{I}$ is converted into K' (as indicated by the arrow in fig. 1) to maintain the eddy motions against the dissipation and export of energy to the zonal mechanical energy. Concerning the magnitude of $C^*(\bar{I},\ \overline{K}+\overline{P})$, we estimate that it is on the order of -0.25 watt m.$^{-2}$ This estimate is based on data analyses by Starr [46] (based on the data by Starr and White [48]); White and Saltzman [51] (over a sector covering the North American continent); and Krueger, Winston, and Haines [15]. This may imply that the direction of conversion $C^*(\bar{I},\ \overline{K}+\overline{P})$ is on the whole from $\overline{K}+\overline{P}$ to $\bar{I}$, but with some uncertainty. The negative sign of C_Z is due to the presence of the middle-latitude indirect meridional circulation. The results of numerical calculations by Phillips [34], Smagorinsky [43], and Smagorinsky, Manabe, and Holloway [44] for C_E and C_Z indicate a reasonably good agreement with observed data. However, the computed values of C_K by the above investigators are roughly two to three times larger than those observed. We shall discuss the results of energetics based on the present model in a separate report.

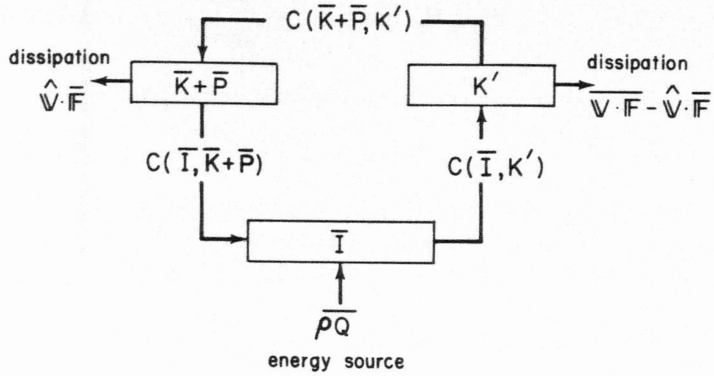

FIGURE 1.—The scheme of energy transformation. The direction of the arrows indicates the "flow" of energy based on various observational data averaged over the Northern Hemisphere and over a period of several months.

A remark should be made about our reason for combining $\overline{K}$ and $\overline{P}$, and not discussing the energy transformation between $\overline{K}$ and $\overline{P}$. The second term on the right-hand side of equation (5.7) represents the rate of work done by the meridional pressure force, and we have the following identity:

$$\frac{\hat{v}}{a}\frac{\partial \bar{p}}{\partial\varphi}=\nabla_3\cdot(\bar{p}\hat{\mathbf{V}}_3)-\bar{p}\nabla_3\cdot\hat{\mathbf{V}}_3-\hat{w}\ \frac{\partial\bar{p}}{\partial z}. \tag{5.24}$$

The second term on the right-hand side of the above equation is $C(\bar{I},\ \overline{K}+\overline{P})$ as already defined by (5.13). Therefore, if we eliminate the $\hat{v}\partial\bar{p}/a\partial\varphi$ term in (5.7) by using (5.24), we find that an energy transformation function connecting $\overline{K}$ and $\bar{I}$ appears in the resulting equation. On the other hand, if we eliminate the $\hat{w}\partial\bar{p}/\partial z$ term in (5.10) using (5.24), then we find that an energy transformation function connecting $\overline{P}$ and $\overline{I}$ appears in the resulting equation. This means that the box of $\overline{I}$ can be connected either to the box of $\overline{P}$ or that of $\overline{K}$, depending upon the way in which the energy equations are written. In the present discussion we have avoided this arbitrariness by combining the two equations (5.7) and (5.10).

Another remark should be made about our reason for introducing only the zonal internal energy $\overline{I}$, and not defining such a quantity as eddy internal energy. As pointed out by Lorenz [20], the partition of kinetic energy into zonal and eddy parts is possible because the kinetic energy is the sum of the variance of the wind components. A similar analysis of variance of the temperature field is also possible. In atmospheric models such as those discussed by Phillips [33, 34] and Smagorinsky [43], the static stability of the atmosphere is treated as a constant parameter. In such systems, the variance of the temperature field is introduced naturally in the course of deriving the total energy equation relevant to the systems. This variance of temperature multiplied by a measure of static stability is the available potential energy defined in [20]. Thus, in a system of constant static stability, the reason for partitioning the available potential energy into the corresponding zonal and eddy parts is evident. However, in a case in which the static stability is not constant, such as the one discussed in this paper, introduction of temperature variance is not required to derive the total energy equation. Thus, in the zonally averaged total energy equation (5.21), a term corresponding to eddy available potential energy does not appear. Of course, one can introduce the concept of available potential energy to derive a new total energy equation in the z-system. We plan to do this in the future following a line somewhat similar to van Mieghem's [26].

6. LINEAR ANALYSIS

As demonstrated by Charney [4], a gross feature of the atmospheric general circulation can be studied by an analysis of a linearized version of the relevant system of atmospheric equations together with the use of energy balance relationships. Since our model is somewhat different from those extensively studied, it is useful to present an analysis of a linearized version of our model.

In order to simplify the mathematical treatment, we adopt the β-plane approximation [36] by taking the two space variables x and y as Cartesian coordinates directed toward the east and the north. The Coriolis parameter f is expressed as $f=f_0+\beta y$ where f_0 is the value of f at the origin of the y coordinate. We ignore the dissipation and heating terms $\mathbf{F}$ and Q.

We express a steady state quantity by a bar placed over the symbol of the quantity. The basic steady state is described by

$$\overline{u}=\overline{u}(y,\ z),\ v=0,\ \overline{w}=0, \tag{6.1}$$

$$\frac{1}{\overline{\rho}}\frac{\partial\overline{p}}{\partial y}+f\overline{u}=0, \tag{6.2}$$

$$\frac{1}{\overline{\rho}}\frac{\partial\overline{p}}{\partial z}+g=0. \tag{6.3}$$

Differentiating (6.2) with respect to z and eliminating $\partial\overline{p}/\partial z$ by using (6.3), we obtain

$$\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial y}=\frac{f\overline{u}}{g}\left(\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial z}+\frac{1}{\overline{u}}\frac{\partial\overline{u}}{\partial z}\right)\equiv G. \tag{6.4}$$

To linearize the system of nonlinear equations, we assume that the motions can be regarded as small perturbations superimposed on the steady states (6.1)–(6.3). Thus, introducing a prime superscript for a perturbation quantity, we have

$$u=\overline{u}(z)+u',\ v=v',\ w=w',\ p=\overline{p}(y,z)+p',\ \rho=\overline{\rho}(y,z)+\rho'. \tag{6.5}$$

Substituting (6.5) into (2.1), (2.2), (2.10), and (2.13) (together with (2.14) and (2.15)) and then dropping quantities of the second and higher order, we obtain the following set of linear equations for the first-order quantities in which the zero-order (steady-state) quantities $\overline{u}$, $\overline{p}$, $\overline{\rho}$, etc., appear as coefficients:

$$L[u']=fv'-w'\frac{\partial\overline{u}}{\partial z}-\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial x}, \tag{6.6}$$

$$L[v']=-fu'-\frac{f\overline{u}}{\overline{\rho}}\rho'-\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial y}, \tag{6.7}$$

$$\frac{\partial p'}{\partial t}=B'+g\overline{\rho}w'-g\int_z^{z_T}\nabla\cdot(\overline{\rho}\mathbf{V}')dz-g\int_z^{z_T}\overline{u}\frac{\partial\rho'}{\partial x}\,dz \tag{6.8}$$

$$\frac{\partial w'}{\partial z}=-\nabla\cdot\mathbf{V}'-\frac{1}{\gamma\overline{p}}\left\{B'+\overline{u}\frac{\partial p'}{\partial x}+v'\frac{\partial\overline{p}}{\partial y}-g\int_z^{z_T}\nabla\cdot(\overline{\rho}\mathbf{V}')dz-g\int_z^{z_T}\overline{u}\frac{\partial\rho'}{\partial x}\,dz\right\} \tag{6.9}$$

where

$$\left.\begin{aligned}\nabla\cdot(\overline{\rho}\mathbf{V}')&=\frac{\partial(\overline{\rho}u')}{\partial x}+\frac{\partial(\overline{\rho}v')}{\partial y}\\ L[\ ]&=\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\end{aligned}\right\}, \tag{6.10}$$

and

$$B'=\frac{\partial p'}{\partial t}\ \text{at}\ z=z_T.$$

The upper and lower boundary conditions are

$$w'=0\ \text{at}\ z=0\ \text{and}\ z=z_T. \tag{6.11}$$

Since the perturbation motions are in hydrostatic equilibrium, we have

$$\rho'=-\frac{1}{g}\frac{\partial p'}{\partial z}. \tag{6.12}$$

We can, therefore, eliminate ρ' from (6.7), (6.8), and (6.9) by using (6.12). However, it turns out that the density perturbation terms can be ignored from the system of equations because the following approximation holds

$$\frac{u'}{\bar{u}} >> \frac{\rho'}{\bar{\rho}}. \tag{6.13}$$

To prove (6.13), let us apply the operator L to (6.7) and eliminate $L[u']$ from the resulting equation using (6.6). The result is

$$L^2[v'] + f^2 v' - w' f \frac{\partial \bar{u}}{\partial z} - \frac{f}{\bar{\rho}} \frac{\partial p'}{\partial x} = -\frac{f\bar{u}}{\bar{\rho}} L[\rho'] - \frac{1}{\bar{\rho}} L\left[\frac{\partial p'}{\partial y}\right]. \tag{6.14}$$

The order of magnitude of the first term I on the right-hand side of (6.14) becomes

$$I \sim \frac{\bar{u}^2}{gH_0} \frac{f}{\bar{\rho}} \frac{\partial p'}{\partial x}$$

by using (6.12) and the approximations $L[\ ] \sim \bar{u}\partial[\ ]/\partial x$ and $\partial p'/\partial z \sim p'/H_0$, where H_0 is the height of a homogeneous atmosphere. Therefore, the magnitude of the first term on the right-hand side of (6.14) is smaller than that of the last term on the left-hand side of (6.14) by the factor of $\bar{u}^2/(gH_0)$, which is on the order of 10^{-3}. Charney [3] has used approximations similar to (6.13) for his analysis of the stability of long waves. Thus, the terms of density perturbation are ignored altogether from the present analysis. Also, by employing a scale analysis presented by Charney [2], it can be shown that the term $w'\partial\bar{u}/\partial z$ in (6.6) can be neglected for large-scale motion.

In order to get some idea how the set of nonlinear equations (2.1), (2.2), (2.10), and (2.16) may be written in the form of corresponding finite-difference equations, we shall consider the simplest model of the atmosphere, i.e., a two-layer model. After some judicious consideration, the perturbation quantities u', v', p', and w' are placed on the atmospheric levels as shown in figure 2. Here the subscripts T, M, and S refer to the top, middle, and surface levels, respectively. The subscripts 2 and 1 refer to the levels at the $(1/2)\Delta z$ and $(3/2)\Delta z$ heights, where Δz is the thickness of either the upper or the lower layer.

Let us introduce the following symbols:

$$\left.\begin{array}{l} \left.\begin{array}{l} m_i = \bar{\rho}_i u_i' \\ n_i = \bar{\rho}_i v_i' \end{array}\right\} \quad i=1, 2 \\ p_1 = (p_M' + p_S')/2 \\ p_2 = (p_T' + p_M')/2 \end{array}\right\}. \tag{6.15}$$

With reference to (6.6) and (6.7) the perturbation momentum equations at levels 1 and 2 are

$$L_i[m_i] - fn_i = -\frac{\partial p_i}{\partial x}, \tag{6.16a}$$

$$L_i[n_i] + fm_i = -\frac{\partial p_i}{\partial y}, \tag{6.16b}$$

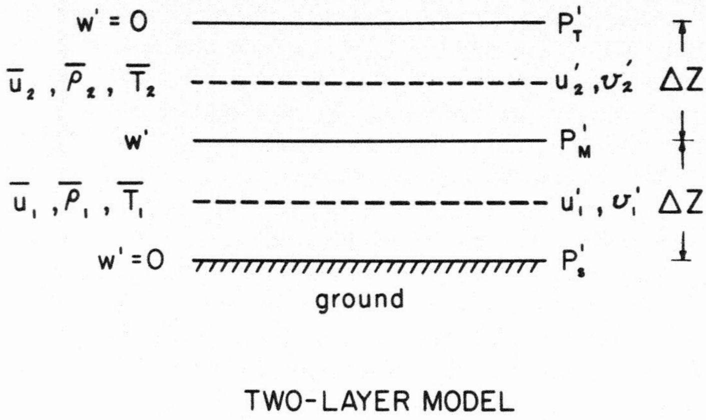

FIGURE 2.—A two-layer model.

where $i=1$, 2 and

$$L_i[\ ] = \frac{\partial}{\partial t} + \bar{u}_i \frac{\partial}{\partial x}. \tag{6.17}$$

With reference to (6.8), the perturbation tendency equations at levels 1 and 2 may be written

$$\frac{\partial p_1}{\partial t} = B' + g\bar{\rho}_M \frac{w'}{2} - g\frac{\Delta z}{2} \nabla\cdot\mathbf{M}_1 - g\Delta z \nabla\cdot\mathbf{M}_2 \tag{6.18}$$

$$\frac{\partial p_2}{\partial t} = B' + g\bar{\rho}_M \frac{w'}{2} - g\frac{\Delta z}{2} \nabla\cdot\mathbf{M}_2 \tag{6.19}$$

where

$$\left.\begin{array}{l} \bar{\rho}_M = (\bar{\rho}_1 + \bar{\rho}_2)/2 \\ \nabla\cdot\mathbf{M}_i = \dfrac{\partial m_i}{\partial x} + \dfrac{\partial n_i}{\partial y} \end{array}\right\}. \tag{6.20}$$

Using the boundary conditions (6.11), equation (6.9) may be expressed in the following finite difference forms at levels 1 and 2:

$$\bar{\rho}_1 \frac{w'}{\Delta z} = -\nabla\cdot\mathbf{M}_1 + n_1 G_1 - \frac{1}{\sigma^2}\left[\bar{u}_1 \frac{\partial p_1}{\partial x} - f\bar{u}_1 n_1 - g\Delta z \nabla\cdot\mathbf{M}_2 - g\frac{\Delta z}{2}\nabla\cdot\mathbf{M}_1 + B'\right] \tag{6.21a}$$

$$-\bar{\rho}_2 \frac{w'}{\Delta z} = -\nabla\cdot\mathbf{M}_2 + n_2 G_2 - \frac{1}{\sigma^2}\left[\bar{u}_2 \frac{\partial p_2}{\partial x} - f\bar{u}_2 n_2 - g\frac{\Delta z}{2}\nabla\cdot\mathbf{M}_2 + B'\right] \tag{6.21b}$$

where $\sigma = (\gamma\bar{p}/\bar{\rho})^{1/2}$ is the Laplacian velocity of sound which is treated as a constant, and the quantity G is defined by (6.4).

The quantities w' and B' can be determined from solving (6.21a, b). After substituting the determined expressions of w' and B' in (6.18) and (6.19), we obtain:

$$\frac{\partial p_1}{\partial t} = A_1 \nabla\cdot\mathbf{M}_1 + A_2 \nabla\cdot\mathbf{M}_2 + D \tag{6.22a}$$

$$\frac{\partial p_2}{\partial t} = B_1 \nabla\cdot\mathbf{M}_1 + B_2 \nabla\cdot\mathbf{M}_2 + D \tag{6.22b}$$

where

$$A_1=-S_2\sigma^2-\frac{g\Delta z}{4}\left(1-\frac{g\Delta z}{2\sigma^2}\right)-S_1\frac{g\Delta z}{2}$$

$$A_2=-S_1\sigma^2+\frac{g\Delta z}{4}\left(1+\frac{g\Delta z}{2\sigma^2}\right)-S_1\frac{g\Delta z}{2}$$

$$B_1=-S_2\sigma^2-\frac{g\Delta z}{4}\left(1-\frac{g\Delta z}{2\sigma^2}\right)+S_2\frac{g\Delta z}{2}$$

$$B_2=-S_1\sigma^2+\frac{g\Delta z}{4}\left(1+\frac{g\Delta z}{2\sigma^2}\right)+S_2\frac{g\Delta z}{2}$$

$$D=\left(\frac{g\Delta z}{4}+S_2\sigma^2\right)\left\{n_1G_1-\frac{1}{\sigma^2}\left(\overline{u}_1\frac{\partial p_1}{\partial x}-f\overline{u}_1n_1\right)\right\}$$
$$-\left(\frac{g\Delta z}{4}-S_1\sigma^2\right)\left\{n_2G_2-\frac{1}{\sigma^2}\left(\overline{u}_2\frac{\partial p_2}{\partial x}-f\overline{u}_2n_2\right)\right\}$$

$$S_i=\frac{\overline{\rho}_i}{\overline{\rho}_1+\overline{\rho}_2}\qquad\text{for } i=1, 2. \tag{6.23}$$

We assume that the perturbation quantities are proportional to exp $ik(x-ct)$ where $i=\sqrt{-1}$, k is the wave number, and c is the phase velocity. The expressions for m_i and n_i can now be obtained from (6.16) as follows:

$$m_i=\frac{k^2(\overline{u}_i-c)p_i-f\frac{\partial p_i}{\partial y}}{f^2-k^2(\overline{u}_i-c)^2},\qquad i=1, 2 \tag{6.24a}$$

$$n_i=\frac{-ik(\overline{u}_i-c)\frac{\partial p_i}{\partial y}+ikfp_i}{f^2-k^2(\overline{u}_i-c)^2}. \tag{6.24b}$$

We now make the following assumptions:

1. All perturbation quantities are independent of y.
2. Perturbation motions are quasi-geostrophic in the sense of Charney [3] and Eady [6]. That is, $f^2>>k^2(\overline{u}-c)^2$.

By making use of the above assumptions, and introducing the expressions of m_i and n_i into (6.22), we obtain two equations for p_1 and p_2. Since the two equations are homogeneous, there exist non-trivial solutions provided that the phase velocity c satisfies the following algebraic equation:

$$Ec^2+Fc+H=0 \tag{6.25}$$

where

$$E=a_1a_2-d_1d_2,$$
$$F=a_1b_2+a_2b_1+h_1d_2+h_2d_1,$$
$$H=b_1b_2-h_1h_2$$

and

$$a_i=1-e_i,\qquad i=1, 2$$
$$e_1=A_1/C_I^2,$$
$$e_2=B_2/C_I^2,$$
$$b_i=K_i+e_i(\overline{u}_i+C_R),\qquad i=1, 2$$
$$d_1=B_1/C_I^2,$$
$$d_2=A_2/C_I^2,$$
$$h_i=K_i+d_i(\overline{u}_i+C_R),\qquad i=1, 2$$
$$K_1=\left(S_2\sigma^2+\frac{g\Delta z}{4}\right)G_1/f_0,$$
$$K_2=\left(S_1\sigma^2-\frac{g\Delta z}{4}\right)G_2/f_0,$$
$$C_I=f_0/k,$$
$$C_R=-\beta/k^2,$$
$$\beta=\partial f/\partial y=\text{Rossby parameter.}$$

We assume that $\overline{u}=\Lambda z$ where Λ denotes the vertical wind shear which is assumed to be a constant parameter and $\Lambda>0$. Thus, $\overline{u}_1=\Lambda(\Delta z/2)$ and $\overline{u}_2=\Lambda(3\Delta z/2)$. For given values of $\overline{\rho}_1$, $\overline{\rho}_2$, g, Δz, f_0, and β, the coefficients E, F, and H of equation (6.25) are functions of only k, the wave number, or $L=2\pi/k$, L being the wavelength, and Λ the vertical wind shear. We see that for prescribed values of L and Λ there are two solutions of c. When c becomes complex, it means that the perturbation either amplifies or damps exponentially. The exponential growth rate is expressed by exp (kc_it) where c_i is the imaginary part of c. The region of instability in the L-Λ parameter domain is found by solving (6.25) as shown in figure 3. In this computation, the following numerical values are used for the parameters:

$\overline{p}_s=1013.3$ mb.,	$g=980$ cm. sec.$^{-2}$,
$\overline{T}_2=229.7$°K.,	$\Delta z=6\times10^5$ cm.,
$\overline{T}_1=268.6$°K.,	latitude$=45$°.

The values of surface pressure $\overline{p}_s$ and temperature $\overline{T}_1$ and $\overline{T}_2$ are taken from the U.S. Standard Atmosphere data. The values of $\overline{\rho}_1$ and $\overline{\rho}_2$ are computed from the joint use of the hydrostatic equation and the gas law for the steady state. In figure 3 the solid lines in the region of instability show the e-folding time in days, which is defined by the inverse of kc_i. The dashed line shows the line of maximum instability. The wavelength of maximum instability appears around 4500 km. This corresponds to a wave number of about six to seven at 45° latitude.

The general characteristics of the instability diagram agree with those of Thompson [49], Phillips [33], and others. As suggested by Kuo's [17] study, the fact that the stable domain appears in a short-wave region may be due to a vertical truncation error caused by approximation of the vertical structure of the atmosphere by a two-layer model. The line for the short-wave cut-off bends toward larger wavelengths as the vertical wind shear increases. Wiin-Nielsen [52] obtained a similar result in his baroclinic stability analysis of a two-layer model using primitive equations of large-scale motions.

The appearance of the stable region for longer wavelengths is due to the stabilizing effect of north-south

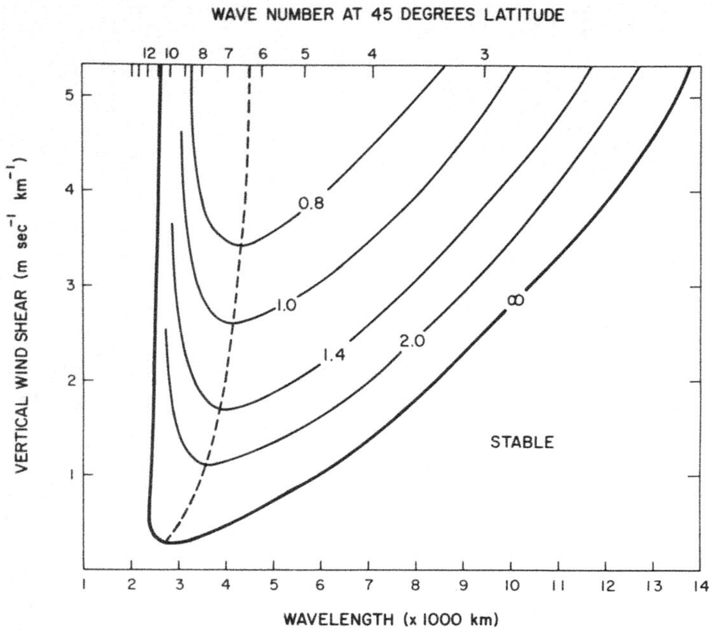

FIGURE 3.—The domain inside the heavy solid line indicated by ∞ is the unstable region. The thin solid lines show the *e*-folding time in days for the growth of unstable waves. The dashed line represents a curve on which the growth rate is maximum.

variations of the Coriolis parameter. A similar stability diagram was constructed (but is not shown here) by setting $\beta=0$ in the calculation, but making no other changes. It was found that again a stable region appears in a short-wave region, but the instability domain has no upper limit in a manner similar to that discussed by Eady [6]. In the case $\beta\neq 0$, the short-wave cut-off appears at about 2500–3000 km., whereas in the case $\beta=0$, the short-wave cut-off appears at about 3500–4000 km. This indicates that the variation of the Coriolis parameter β has a destabilizing effect in the short-wave regime, as pointed out by Green [12].

In this calculation, we assumed westerlies as the basic zonal flow. However, a similar stability diagram was constructed (but not shown here) by assuming that $\overline{u}_2=\Lambda(\Delta z/2)$ and $\overline{u}_1=-\overline{u}_2$ in the model, but making no other changes. In this case, the lower basic flow is easterly and the upper one is westerly, and the average of the two vanishes. The short-wave cut-off appears, in this case, at 1500–2000 km. in wavelength and the maximum instability appears at approximately 2500 km. These results turn out to be very similar to those obtained by Fjørtoft [9]. Since the stability characteristics of the present model are in good agreement with those obtained by previous investigators, we feel that the present formulation for the linear two-layer version of our model is applicable to the finite-difference formulation for the full nonlinear equations.

7. REMARKS

A test of this model was performed with a two-layer version of the model using a vertical spacing of 6 km. The grid network covers the entire globe with a horizontal spacing of 5° in both longitude and latitude. Near the poles the grid is coarsened in the longitudinal direction in order to keep the geographical distances more uniform. Finite-difference equations for the model are formulated based on a modified version of the two-step Lax-Wendroff scheme proposed by Richtmyer [40]. A time step of 5 min. was used.

The effect of continentality is reflected in the distribution of sea level temperature, which is prescribed in the input data, but orography is ignored. An isothermal atmosphere of 250° K. is used as the initial condition. The sea level temperature and the declination of the sun are prescribed for mean January conditions. Typical Highs and Lows are generated after approximately 30–40 days, resulting from baroclinic instability.

At present many long-term (over 100 days) numerical integrations are being made on the CDC 6600 by changing the magnitudes of various physical parameters within their reasonable ranges. It is important to change only one parameter at a time to see the response of varying the parameters of the model. The results of integrations are compared with the climatological data of the atmosphere. Comparison data include zonal mean distribution of wind velocity components, temperature, and pressure in a meridional cross section, etc. Details of the finite-difference methods and the results of numerical calculations will be described in a separate report.

ACKNOWLEDGMENTS

This paper describes the formulation of a model of the atmospheric general circulation. The value of a paper of this kind is lessened if the formulation is not supported by actual experimentation. As mentioned in the previous section, the entire formulation has been programed for the CDC 3600 and 6600 computers and tested. In addition to use of the computer, the NCAR Computing Facility provided a cathode ray visual display device called the dd 80. This device was used very effectively to display the global distribution of all of the dependent variables for presentation as well as for debugging the program. It is noteworthy to add here that our progress would have been seriously hindered without it.

It is also pertinent to acknowledge the members of the NCAR Computing Facility for their effective and imaginative assistance. We would like to point out particularly the aid which Bernard O'Lear furnished with systems and utility programing. Joyce Takamine, Larry Williams, and Dave Robertson wrote many of the dd 80 programs necessary for the visual display. The head computer operators Ben Klepac and Don Austin supervised effectively the many hours of running time on the computers.

Placido Jordan prepared the map of global surface

temperature distribution for a mean January which was used in the program as part of the input data and prepared many of the graphs to be presented in the second part of the paper which will appear later.

A large research undertaking such as this one inevitably provides a scientific conversational piece among the scientific visitors to NCAR, as well as among our own colleagues. In addition to those mentioned in the text, many useful ideas in such diverse areas as the formulation of the problem to advice about the organization of the program for the computer were drawn from many hours of discussion with those researchers. If we were to list the names of those people it would include practically all of the major research workers in the field of dynamic and synoptic meteorology both in the United States and abroad. Finally, but not least, Kathy Hatt assisted in preparation of this manuscript.

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